

# Spin-It: Optimizing Moment of Inertia for Spinnable Objects

Moritz Bächer  
Bernd Bickel

Disney Research Zurich

Emily Whiting  
Olga Sorkine-Hornung

ETH Zurich

## This is an *easy-read* summary of a **SIGGRAPH2014** Technical Paper

As part of our annual NoirMaker social game, we aim to make some of the Conference more accessible to our international audience. For 2014, we have summarized and translated a few of the Conference's Technical Papers, to serve as catalyst for 'technical' conversations that will be part of our regional presentations, including:

*'CG in Australasia'*

Sunday, August 10, 1030 am

in the ACM SIGGRAPH Theater | International Center  
[room 210, West Building; VCC]

We hope that this will inspire you to access so many more interesting Technical Papers that will be presented during the Conference week in Vancouver:

<http://bit.ly/s2014-technical-papers>

And remember that Technical Papers can be found after the Conference, as part of the ACM Digital Library:

<http://dl.acm.org>

Brought to you by:



Spinning tops and yo-yos have long fascinated cultures around the world with their unexpected, graceful motions that seemingly elude gravity. We present an algorithm to generate designs for spinning objects by optimizing rotational dynamics properties. As input, the user provides a solid 3D model and a desired axis of rotation. Our approach then modifies the mass distribution... We [also] augment the model by creating voids inside its volume... We extend our technique to incorporate deformation and multiple materials for cases where internal voids alone are insufficient. Our method is well-suited for a variety of 3D printed models, ranging from characters to abstract shapes. We demonstrate tops and yo-yos that spin surprisingly stably despite their asymmetric appearance...

[4 basic requirements for the model to spin stably:]

Given a 3D shape, we aim to generate spinnable models by altering their mass properties, while keeping the appearance as close to the original as possible.

### *User Input*

The user provides the surface of a solid 3D shape, along with the desired spinning axis  $\mathbf{a}$ . The axis origin is set to the contact point  $\mathbf{p}$ ... For yo-yo designs, the shape is partitioned into two parts and connected with an axle that aligns with  $\mathbf{a}$ ...

### *Mass Properties and Constraints*

We denote by  $\mathbf{M}$  the mass of our object and by  $\mathbf{c}$  the center of mass. If we assume a frictionless spin, the only external torque acting on a spinning top relative to  $\mathbf{p}$ , is the gravitational torque with magnitude  $|\boldsymbol{\tau}| = \mathbf{M}\mathbf{g}\mathbf{d}$ , where  $\mathbf{g}$  is Earth's gravity and  $\mathbf{d}$  is the distance from  $\mathbf{c}$  to the spinning axis... During the spinning motion, the precession angle  $\boldsymbol{\theta}$ ... increases... We can express the gravitational torque as  $|\boldsymbol{\tau}| = \mathbf{M}\mathbf{g}\boldsymbol{\ell}\sin\boldsymbol{\theta}$ , where  $\boldsymbol{\ell}$  is the height of the center of mass. Hence, we expect a longer, more stable spin for smaller values of  $\boldsymbol{\ell}$  and  $\mathbf{M}$ . For yo-yos, the gravitational torque remains zero...

### *Moment of Inertia*

... We can only spin about an axis with constant angular velocity if it is a principal axis of inertia... Due to symmetry,  $\mathbf{E}$ 's principal axes of inertia are parallel to its half-axes, and the corresponding moments  $\mathbf{I}_a$ ,  $\mathbf{I}_b$ , and  $\mathbf{I}_c$  each equal the sum of squares of the two other halfaxes' lengths... Hence, the maximal principal axis of inertia corresponds to the shortest axis  $\mathbf{h}_c$ ...

Refers to a body's behavior under small disturbances... Given three distinct values for the principal moments of inertia... rotation is stable under small perturbations only about the largest and the smallest axis... the stability limit depends on the height of the center of mass  $\ell$  and the mass  $M$  itself: the lower the centroid and the smaller the mass, the less angular velocity  $\omega$  is required for a stable spin... Similarly, we need a smaller  $\omega$  the higher the absolute difference between the largest moment and the mid-moment...

We turn models into spinnable objects by altering their mass properties while keeping the appearance as close to the original as possible. Our primary focus is redistributing mass by hollowing the interior with precisely shaped voids.

### *Hollowing*

We first optimize the shape's mass distribution by introducing inner voids... Cage-based deformation. ... Hollowing is [not] effective for many models... In these instances, we further manipulate mass by introducing deformation optimization. We automatically extract a cage... and deform both the surface and the interior voids...

### *Dual-Density Optimization*

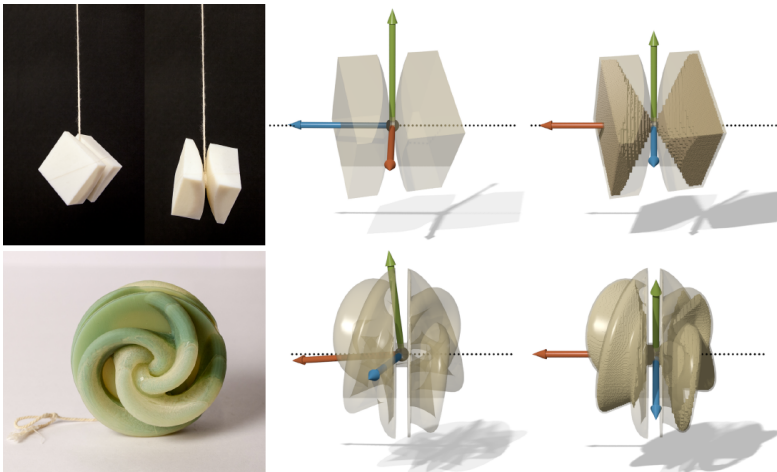
As an alternative to deformation,... a heavier material is used in the interior... While this technique presents an additional fabrication effort, its benefit is avoiding changes to the exterior shape...

... We validated our approach by designing and fabricating a variety of spinning tops... We [also] designed and fabricated two yo-yo examples: the Cuboid... [and the] Woven Ring...

spinning tops



yo-yos: Cuboid and Woven Ring



Get the complete paper:

<http://bit.ly/s2014-paper-spin>

Catch the SIGGRAPH 2014 presentation:

Wednesday, 13 August; 9-1030 am  
[ballroom A, East Building; VCC]

ACM Transactions on Graphics, Vol. 33, No. 4, Article 96, Publication Date: July 2014

© 2014 ACM SIGGRAPH. This is an abstract of the author's version of the work. It is posted here by permission of the ACM SIGGRAPH for your personal use. Not for redistribution.

Permission to make digital or hard copies of part or all of this work for personal or classroom use is granted without fee provided that copies are not made or distributed for profit or commercial advantage and that copies bear this notice and the full citation on the first page. Copyrights for components of this work owned by others than ACM SIGGRAPH must be honored. Abstracting with credit is permitted. To copy otherwise, to republish, to post on servers or to redistribute to lists, requires prior specific permission and/or a fee.